Workshops:
The heart of the MagiKats Programme

Every student is assigned to a Stage, based on their academic year and assessed study level.

Stage 5 students have completed all Stage 4 materials.

The sheets in this pack are a small sample of what is available! These are only samples of the student's worksheets - our teaching methods include discussion and hands-on activities.

Core skills sheets are also provided for independent completion by each student (usually at home).

At this level, students study all the more advanced topics needed to take them to about age 16. Stage 5 material follows on from that in Stage 4 and is offered from age 14. It is only completed by those expecting to continue studying mathematics to a more advanced level.

Maths Stage 5

MagiKats
TUITION CENTRES

Circles

Circle Theorem

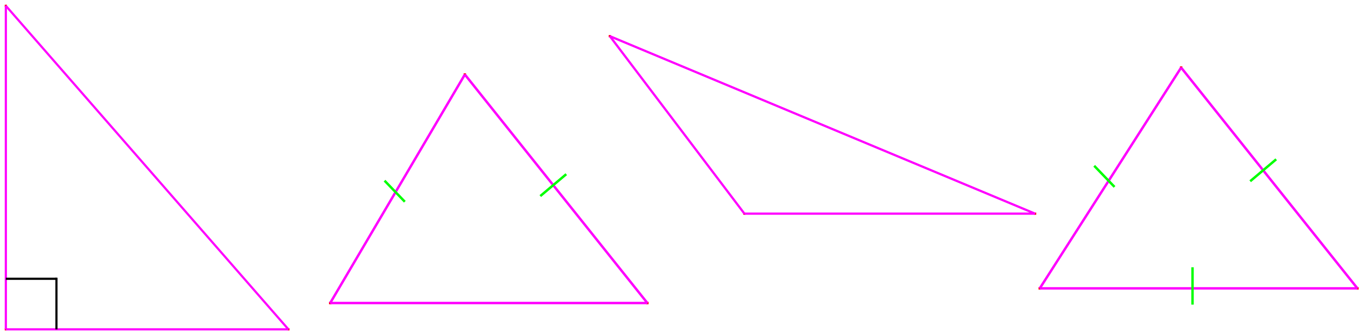
Circle Theorems are all about the different ways in which circles and lines behave when combined. A theorem is basically a statement which has proved to be consistently true.

You could say that circle theorems are just "rules" that can be applied in solving problems involving circles and lines.

Before looking at these rules let us review some basic geometrical facts, and terminology frequently used in circle type problems.

Triangles

Identify the triangles shown below and complete the table.



Name	Number of equal sides	Number of equal angles	Sum of internal angles
			180°
	2		
		0	
Equilateral			

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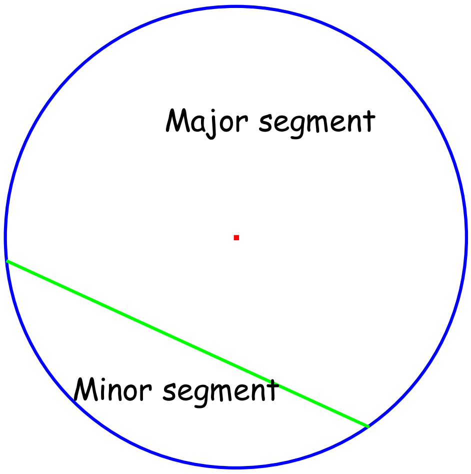
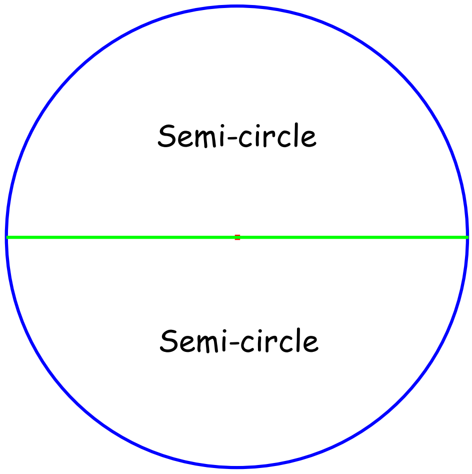
Circles

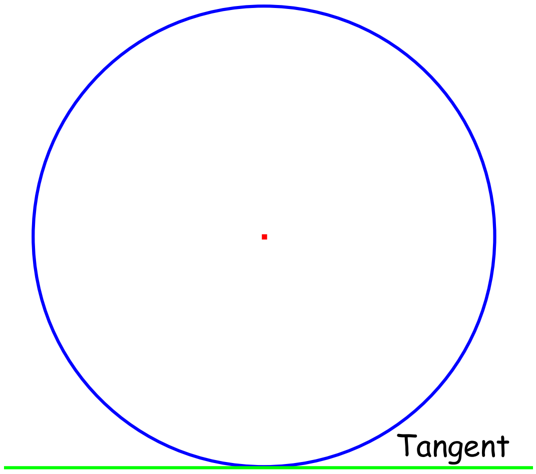
Parts of a circle



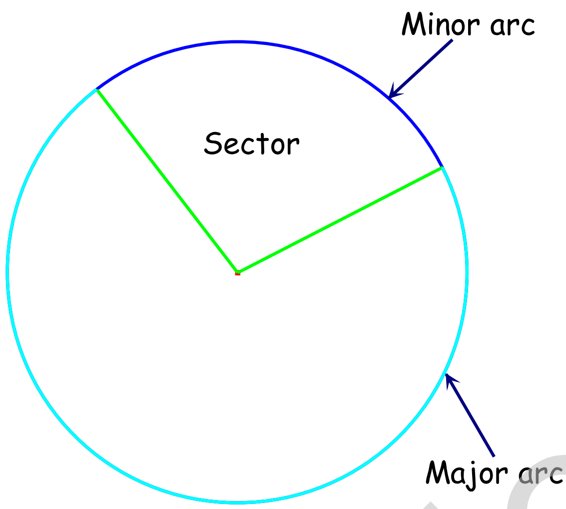
A chord is a straight line that joins two points on the circle, but does NOT pass through the origin. The chord cuts the circle into two segments.



A straight line that joins two points on the circle and passes through the origin is known as the diameter. A diameter cuts the circle into two semi-circles.



A tangent to a circle, is a straight line that touches the outside of a circle at a single point only.



A sector of a circle is the portion of the circle enclosed by two radii and the arc formed by the section of the circumference lying between the two radii.

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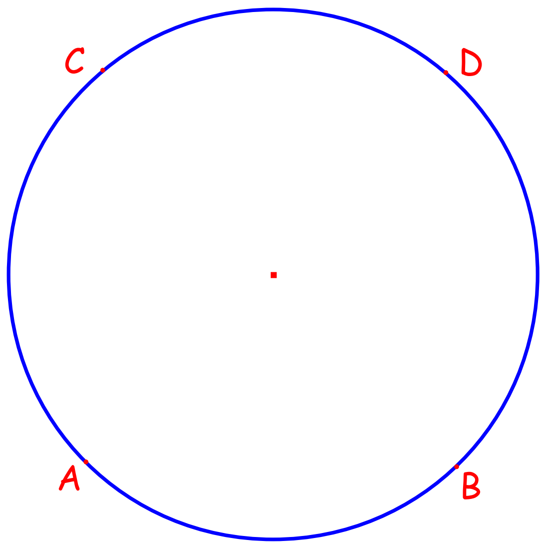
Circles

A set of "rules" have been formulated from observations of different combinations of circles, lines and the angles formed between them. Let us look at some of the more common rules.

Angles in the same segment



What does this mean?

On the circle draw a chord joining the points A and B. Highlight the major arc on the circle and label the major segment.

Using a ruler and a sharp pencil, draw a straight line, joining point A to point C then another line to join C with B.

Now join up A to D and D to B.

As you can see the lines have formed 2 angles, ACB and ADB, both inside the circle on the major arc.

Measure the angles. What do you notice?

You should find that both angles are the same size, depending on how accurately you drew your lines and measured the angles.

Using points A, B, C and D draw two equal angles inside the circle on the minor arc. Which two points would they originate from?

____

Rule 1: Angles in the same segment are equal.
alternative phraseology
Angles standing on the same arc are equal.

All angles in the same segment, originating from the same two points on the circumference of a circle, are equal in size.

Factorisation

Factorising Quadratic Equations

Warm up questions: expand and simplify.

1) $(x + 2)(x + 4)$

2) $(a + 6)(a - 7)$

3) $(s - 6)(s + 11)$

4) $(5y - 3)(y - 5)$

5) $(7t + 6)(5t + 8)$

6) $(x - 3)(3x - 2)$

7) $(2a + 3)(a - 8)$

8) $(2x + 1)(x + 5)$

9) $(3x - 2)(x - 4)$

10) $(x - 5)(4x + 2)$

Any expression of the form $ax + b$, where a and b are numbers, is called a **linear expression in x.**

Look at your answers and you will see that when two linear expressions in x are multiplied, the result usually contains three terms: a term in x^2, a term in x and a number.

Expressions of this form, i.e. $ax^2 + bx + c$ where a, b and c are numbers and $a \neq 0$, are called **quadratic expressions in x.**

Knowing that the product of two linear brackets is quadratic, we should be able to work backwards to factorise a quadratic. For instance, given a quadratic such as $x^2 - 5x + 6$, we could try to find two linear expressions in x whose product is $x^2 - 5x + 6$.

To be able to do this we need to understand the relationship between what is inside the brackets and the resulting quadratic.

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Factorisation

Looking at the last three questions in your warm up:

$$(2x + 1)(x + 5) = 2x^2 + 11x + 5 \quad (1)$$

$$(3x - 2)(x - 4) = 3x^2 - 14x + 8 \quad (2)$$

$$(x - 5)(4x + 2) = 4x^2 - 18x - 10 \quad (3)$$

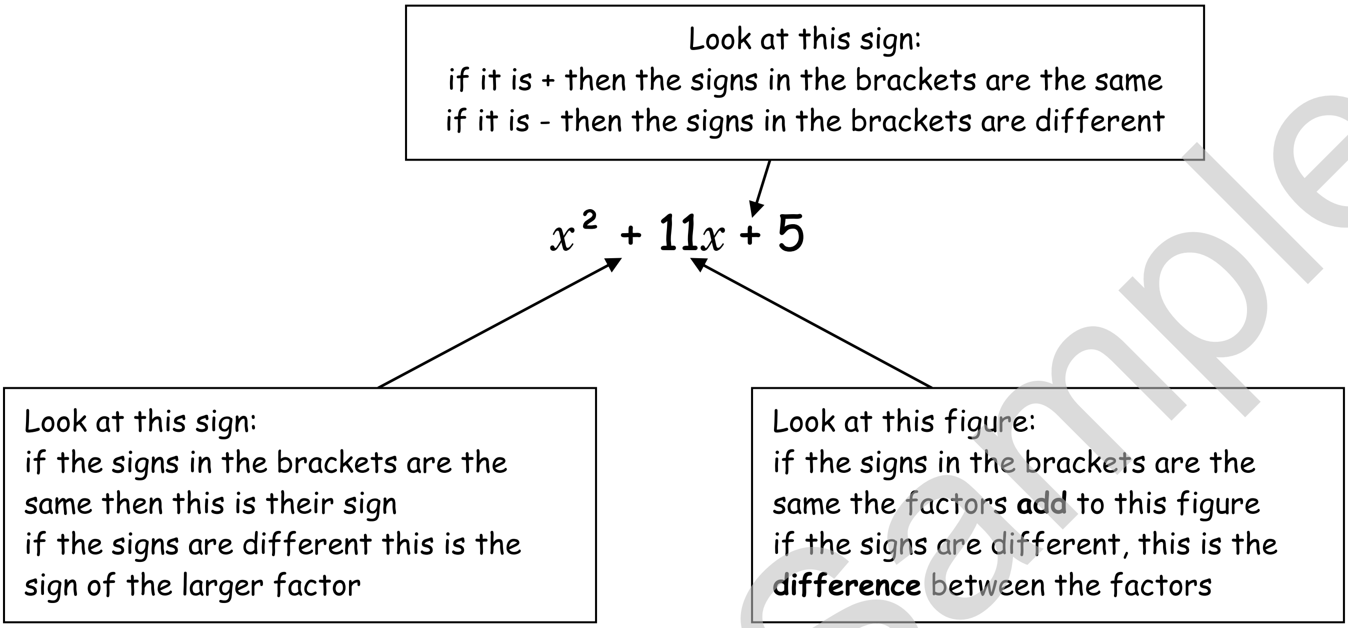
The first thing to notice about the quadratic in each example is that:

- The coefficient of x^2 is the product of the coefficients of x in the two brackets.
- The number term is the product of the numbers in the two brackets.
- The coefficient of x is the sum of the coefficients formed by multiplying the x term in one bracket by the number term in the other bracket.

The next thing to notice is the relationship between the signs.

- + signs throughout the quadratic come from + signs in both brackets, as in (1).
- A + number term and a - coefficient of x in the quadratic come from a - sign in each bracket, as in (2).
- A - number term in the quadratic comes from a - sign in one bracket and a + sign in the other, as in (3).

This ties in neatly with information that you already know: when multiplying two numbers, if their signs are the same then the result is positive but if the signs are different, the result is negative.



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Factorisation

Factorising Quadratic Equations: another way of looking at it!

$(x + a)(x + b) = x^2 + (a+b)x + ab$

Therefore

1) $x^2 + (a+b)x + ab = (x + a)(x + b)$

2) $x^2 - (a+b)x - ab = (x + a)(x - b)$ (where b is the larger digit)

3) $x^2 + (a+b)x - ab = (x - a)(x + b)$ (where b is the larger digit)

4) $x^2 - (a+b)x + ab = (x - a)(x - b)$

Examples

1) $x^2 + 7x + 10 = (x + 2)(x + 5)$

2) $x^2 - 3x - 10 = (x + 2)(x - 5)$

3) $x^2 + 3x - 10 = (x - 2)(x + 5)$

4) $x^2 - 7x + 10 = (x - 2)(x - 5)$

When factorising a quadratic in the form $x^2 + (a+b)x + ab$:

Look at the last term – is it +ve or –ve?

- If it is +ve then the signs in the two brackets are the same – either both + or both -. Their sign is the same as the sign of the x term and the sum of the factors (a+b) is the coefficient of x.

- If it is –ve then the signs are different – one is + and one is -. The larger factor has the same sign as the x term and the difference between the factors (a-b) is the coefficient of x.

Examples: factorising $x^2 + 3x - 10$

last term is –ve so different signs $= (\quad + \quad)(\quad - \quad)$
and factors of 10 (1x10, 2x5) must have a difference of 3
factors for use must be 2x5 and larger one is +ve so $= (x + 5)(x - 2)$

Examples: factorising $x^2 + 11x + 10$

last term is +ve so signs both same as x term $= (\quad + \quad)(\quad + \quad)$
and factors of 10 (1x10, 2x5) must add to 11
factors for use must be 1x10 so $= (x + 1)(x + 10)$

Factorisation

Examples:

1) Factorise $x^2 - 5x + 6$

The x term in each bracket is x as x^2 can only be $x \times x$.
The + sign at the end means that the signs are the same in each bracket
The - sign means both signs are -, so $x^2 - 5x + 6 = (x - \quad)(x - \quad)$
The numbers in the brackets (factors of 6) could be 6 and 1 or 2 and 3.
The signs are the same so add to 5, i.e. the numbers must be 2 and 3.

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Expanding the brackets back checks that they are correct.

2) Factorise $x^2 - 3x - 10$

The x term in each bracket is x
The - sign at the end means that the signs are different $\Rightarrow x^2 - 3x - 10 = (x - \quad)(x + \quad)$
The numbers could be 10 and 1 or 5 and 2.
The signs are different so we need a difference of 3, i.e. they are 5 and 2.

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

Expanding the brackets back checks that they are correct.

Factorise, and then check your answer.

1) $x^2 + 8x + 15$

2) $x^2 + 7x + 6$

3) $x^2 - 10x + 9$

4) $x^2 + 8x + 12$

5) $x^2 + 5x - 14$

6) $x^2 - 4x - 5$

7) $x^2 + 9x + 14$

8) $x^2 - 9$

9) $x^2 + 4x + 4$
think about how to write this answer!

10) $x^2 - 3x - 18$

11) $x^2 - 16$

12) $2x^2 - 3x + 1$

13) $9x^2 - 6x + 1$

14) $9 + 6x + x^2$

Student: Date:

Factorisation

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Extension Work: Harder Factorising

When the number of possible combinations of terms for the brackets increases, common sense considerations can help to reduce the possibilities.

For example, if the coefficient of x in the quadratic is odd, then there must be an even number and an odd number in the brackets.

Example:

Factorise $12 - x - 6x^2$

The x terms in the brackets could be $6x$ and x, or $3x$ and $2x$, one positive and the other negative.

The number terms could be 12 and 1 or 3 and 4 (not 6 and 2 because coefficient of x in the quadratic is odd).

Now we try various combinations until we find the correct one.

$$12 - x - 6x^2 = (3 + 2x)(4 - 3x)$$

Factorise:

1) $6x^2 + x - 12$

2) $4x^2 + 3x - 1$

3) $4x^2 - 12x + 9$

4) $25x^2 - 16$

5) $5x^2 - 61x + 12$

6) $3 + 2x - x^2$

7) $1 - x^2$

8) $x^2 + 2xy + y^2$

9) $4x^2 - 4xy + y^2$

10) $36 + 12x + x^2$

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Direct and Inverse Proportion

Direct and Inverse Proportion

When working on proportion at a higher level, we usually express it in terms of unknowns. Direct proportion is very straight forward. One variable (e.g. y) can be calculated as the other variable (e.g. x) multiplied by some constant. An example would be $y = 3x$

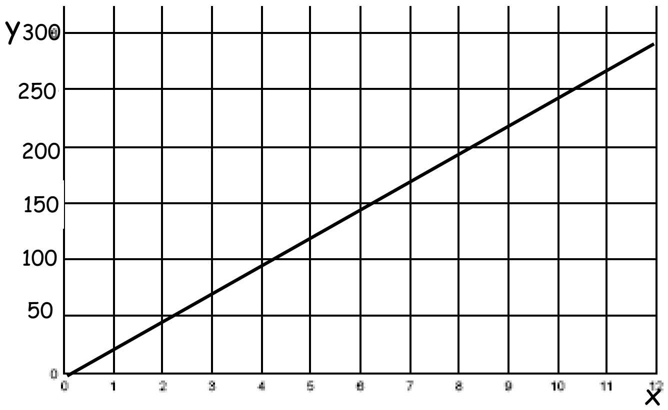
Inverse proportion is a little more complicated. Here, one variable (e.g. y) can be calculated as one over the other variable (e.g. $1/x$) multiplied by some constant.

An example would be $y = 5/x$

Direct Proportion: $y = kx$

Both Increase Together

The graph of y against x is a straight line through the origin: $y = kx$



In both cases k is a constant.

In a table of values the multiplier is the same for x and y, so if you double one of them, you double the other, and so on.

x	2	6	8	12	14
y	3	9	12	18	21

(2 → 6: ×3; 3 → 9: ×3; 6 → 12: ×2; 9 → 18: ×2)

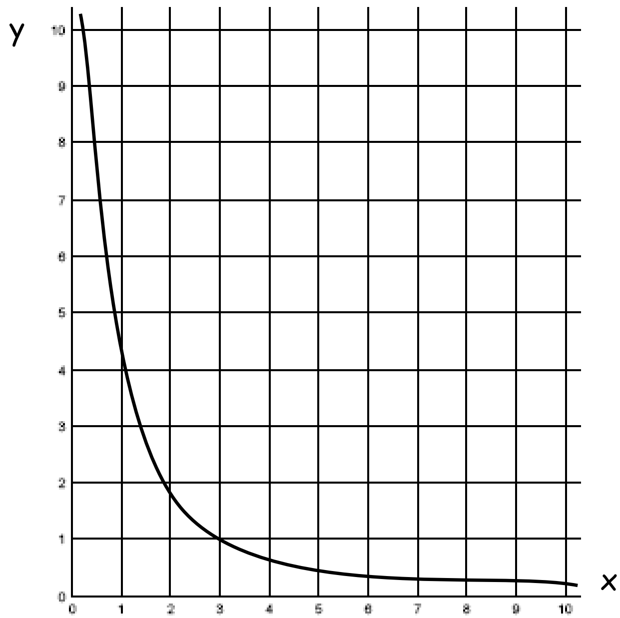
The **RATIO** x/y is the same for all pairs of values.

$$\frac{2}{3} = \frac{6}{9} = \frac{8}{12} = \frac{12}{18} = \frac{14}{21} = 0.6667$$

Inverse Proportion: $y = k/x$

One Increases, One Decreases

Sketch graph of $y = k/x$



In a table of values the multiplier for one of them becomes a divider for the other, so if you double one, you half the other etc.

x	2	6	8	12	40
y	30	10	7.5	5	1.5

(2 → 6: ×3; 30 → 10: ÷3; 6 → 12: ×2; 10 → 5: ÷2)

The **PRODUCT** xy (x times y) is the same for all pairs of values.

2x30= 6x10= 8x7.5= 12x5= 40x1.5= 60

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Direct and Inverse Proportion

Variation

Sometimes your questions may be phrased like these:

"y is proportional to the square of x" "t is proportional to the square root of h"

"$\mathcal{D}$ varies with the cube of t" "$\mathcal{V}$ is inversely proportional to r cubed"

There is a method you must remember to successfully deal with these sorts of questions:

Method

1) **Convert the sentence into a proportionality** using the symbol " $\propto$ " which means "is proportional to".

2) **Replace " $\propto$ " with "=k"** to make an **equation**:

The above examples would become:	Proportionality	Equation
y is proportional to the square of x	$y \propto x^2$	$y = kx^2$
t is proportional to the square root of h	$t \propto \sqrt{h}$	$t = k\sqrt{h}$
$\mathcal{D}$ varies with the cube of t	$\mathcal{D} \propto t^3$	$\mathcal{D} = kt^3$
$\mathcal{V}$ is inversely proportional to r cubed	$\mathcal{V} \propto 1/r^3$	$\mathcal{V} = k/r^3$

3) **Find a pair of values of x and y** somewhere in the question, and **substitute them into the equation.**

4) **Calculate the value of k and put it into the equation** and it's now ready to use.
Example: $y = 3x^2$

5) Finally, find either x or y (depending on which value is already supplied in the question).

Example:

The time taken for a cat to crawl through a drain pipe is inversely proportional to the square of the diameter of the pipe. If he took 25 seconds to crawl through a pipe of diameter 0.3m, how long would it take him to get down one of 0.2m diameter?

Solution:

1) Write it as **proportionality**, then an **equation**: $t \propto 1/d^2$ $t = k/d^2$
2) **Substitute the given values** for the two variables: $25 = k/0.3^2$
3) Rearrange the equation to **find k**: $k = 25 \times 0.3^2 = 2.25$
4) **Put k back in** the formula: $t = 2.25/d^2$
5) **Substitute in the new value** for d: $t = 2.25/0.2^2 = $ 56.25secs

Direct and Inverse Proportion

Scale Factors

Scale factors and proportion are used every day to represent small or larger objects, shapes or images.

Typical examples of these would be:

A medium sized wall map of the World

Scale: 1:30,000,000 which represents 1cm to 300km

A road map for motorists

Scale: 1: 250,000 which represents 1 cm to 2.5km

An Ordnance survey map for walkers or hikers

Scale: 1:25,000 which represents 1cm to 250m

An architects drawing

Scale: 1:100 which represents 1cm to 1m

These questions are straight forward but can look off putting. Assume these are straight forward and give them a try!

1) A sports hall will have a main gymnasium that is 20m long. The architect uses a scale of 1:100. How long will the gym be on his plan?

2) A map has a scale of 1cm : 3 miles. On the map, two towns are 7cm apart. What is the actual distance between the two towns?

3) A map has a scale of 1cm represents 50 metres.

a) Which ratio is equivalent to this?

1:50 1:500 1:5000 1:50000 1:500000 1:5000000

b) The distance between two houses on the map is 4.5cm. What is the actual distance between them?

Now this one – take special care with part c.

4) A map is drawn with the scale 1:25 000.

a) a distance on the map is 5cm. What is the real distance?

b) the length of a road is 5km. How long will it be on the map?

c) the area of forest is 10 cm^2 on the map. What is its actual area?

Indices and Surds

Index notation

Index notation is used to represent powers, for example

a^2 means a × a and here the index is 2

b^3 means b × b × b and here the index is 3

c^4 means c × c × c × c and here the index is 4 etc.

2 x 2 x 2 x 2 = 16 $2^4 = 16$	• 2 is called the base • 4 is called the index • 16 is called the basic numeral

$x^n = \underbrace{x \times x \times x \times x \times \ldots\ldots x \ x \times x}_{n \text{ factors}}$ (where n is a positive integer)

For: x^n — x is the base, n is the index.

There are several "laws" about index numbers and how they work. We will go through them, one by one. Provided that you understand them then they are easy and you may find that don't even need to memorise them.

Multiplication using indices

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$
$= 3^6 \quad [=3^{4+2}]$

$x^5 \times x^3 = (x \times x \times x \times x \times x) \times (x \times x \times x)$
$= x^8 \quad [= x^{5+3}]$

Law 1: When multiplying terms, *add* the indices: $x^m \times x^n = x^{m+n}$

Division using indices

$3^5 \div 3^2 = \frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}}$
$= 3^3 [= 3^{5-2}]$

$x^4 \div x^3 = \frac{x \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x}}$
$= x^1 [= x^{4-3}]$

Law 2: When dividing terms, *subtract* the indices: $x^m \div x^n = x^{m-n}$

Student: Date:

Indices and Surds

Powers of indices

$(3^3)^2 = 3^3 \times 3^3$
$= 3^{3+3}$ [Using Law 1]
$= 3^6$ [$= 3^{3 \times 2}$]

$(x^5)^4 = x^5 \times x^5 \times x^5 \times x^5$
$= x^{5+5+5+5}$ [Using Law 1]
$= x^{20}$ [$= x^{5 \times 4}$]

Law 3: For powers of a power, *multiply* the indices: $(x^m)^n = x^{mn}$

If we simplify the division $x^n \div x^n$, using the second law above:

$x^n \div x^n = x^{n-n}$
$= x^0$

But any expression divided by itself must equal 1. $x^n \div x^n = 1$

Therefore x^0 must be equal to 1. $x^0 = 1$

Law 4: $x^0 = 1$

1) Simplify:
a) 4^3 b) 13^5 c) $(-4)^2$

2) Simplify:
a) $3^2 \times 3^5$ b) $x^3 \times x^2$ c) $6m^2n \times mn^4$

3) Simplify:
a) $x^7 \div x^2$ b) $15a^5 \div 3a^2$ c) $20a^3b^2 \div 10ab$

4) Simplify:
a) $(a^4)^2$ b) $(2a^4)^3$ c) $(p^4)^3 \div (p^2)^4$

5) Simplify:
a) 7^0 b) $18x^3 \div 6x^3$ c) $(2y^3)^4 \div (4y^6)^2$

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Indices and Surds

<u>Negative Indices</u>

All the indices seen so far have been positive integers or zero.

If we had $2^3 \div 2^5$, the answer, according to the second index law, should be 2^{3-5},
ie $2^3 \div 2^5 = 2^{-2}$.

But this could also be written in this way:

$$\frac{2^3}{2^5} = \frac{\cancel{2}^1 x \cancel{2}^1 x \cancel{2}^1}{\cancel{2}^1 x \cancel{2}^1 x \cancel{2}^1 x 2 x 2}$$

$$= \frac{1}{2x2}$$

$$= \frac{1}{2^2}$$

So $2^{-2} = \frac{1}{2^2}$

In general, the meaning of a negative index can be summarised by the rules:

$$x^{-m} = \frac{1}{x^m} \quad , \ (x \neq 0)$$

x^{-m} is the reciprocal of x^m, since x^m x x^{-m} = 1.

Explanation: when you see a minus sign in front of a power, it means that you can rewrite it as 1 over the expression with the minus sign removed.

1) Simplify the following:

a) 3^{-2} b) 5^{-1} c) x^7 x x^{-3}

d) $6x^2 \div 3x^4$ e) $\left(\frac{1}{4}\right)^{-2}$ f) $\left(\frac{2}{3}\right)^{-3}$

2) Evaluate, using a calculator:

a) 2^{-3} b) $\left(\frac{1}{3}\right)^{-2}$

Indices and Surds

Surds

Find the value of:

1) $\sqrt{16}$ 2) $\sqrt{9}$ 3) $\sqrt{36}$ 4) $\sqrt{16+9}$ 5) $\sqrt{16}+\sqrt{9}$

6) $\sqrt{16 x 9}$ 7) $\sqrt{16} x \sqrt{9}$ 8) $\sqrt{\frac{36}{9}}$ 9) $\frac{\sqrt{36}}{\sqrt{9}}$ 10) $(\sqrt{16})^2$

Surds are numerical expressions that involve irrational roots. They are irrational numbers.

Surds obey the following rules.

Rule 1 $\sqrt{xy} = \sqrt{x} \text{ x } \sqrt{y}$

Worked examples:

1) $\sqrt{100} = \sqrt{4} x \sqrt{25}$
$= 2x5$
$= 10$ (which is true)

2) $\sqrt{27} = \sqrt{9} x \sqrt{3}$
$= 3x\sqrt{3}$
$= 3\sqrt{3}$

3) $\sqrt{5} x \sqrt{7} = \sqrt{5x7}$
$= \sqrt{35}$

Rule 2 $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Note: $\sqrt{x}$ means *the positive square root of* x when $x > 0$.

$\sqrt{x} = 0$ when x = 0.

Worked examples:

1) $\sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}}$
$\sqrt{4} = \frac{4}{2}$
$= 2$ (which is true)

2) $\sqrt{125} \div \sqrt{5} = \sqrt{125 \div 5}$
$= \sqrt{25}$

3) $\sqrt{30} \div \sqrt{5} = \sqrt{30 \div 5}$
$= \sqrt{6}$

Rule 3 $(\sqrt{x})^2 = x$ Note: For $\sqrt{x}$ to exist, x cannot be negative.

Worked examples:

1) $(\sqrt{25})^2 = (5)^2$
$= 25$

2) $(\sqrt{7})^2 = 7$

3) $(3\sqrt{2})^2 = 3^2 x (\sqrt{2})^2$
$= 9x2$
$= 18$

Indices and Surds

A surd is in its simplest form when the number under the square root sign is as small as possible. To simplify a surd we make use of Rule 1 by expressing the square root as the product of two smaller square roots, one being the root of a square number. Examine the examples below.

1) Simplify the following surds. The first part of each question has been done.

a) $\sqrt{18} = \sqrt{9} x \sqrt{2}$ b) $\sqrt{75} = \sqrt{25} x \sqrt{3}$ c) $5\sqrt{48} = 5x\sqrt{16} x \sqrt{3}$

Now try these:

Express in terms of the simplest possible surd.

2) $\sqrt{12}$

3) $\sqrt{48}$

4) $\sqrt{200}$

Expand and simplify where this is possible.

5) $\sqrt{3}(2-\sqrt{3})$

6) $\sqrt{2}(5+4\sqrt{2})$

7) $(\sqrt{5}-3)(2\sqrt{5}-4)$

8) $(4+\sqrt{7})(4-\sqrt{7})$

Sine Rule

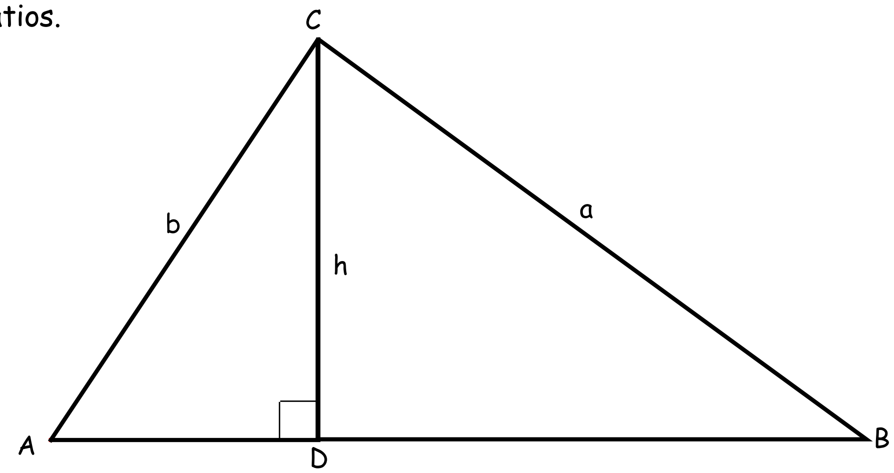
The Sine Rule

Finding unknown angles and sides in right angled triangles is straight forward. We can use sin, cos, tan or Pythagoras. Sadly, not all triangles are right angled. The next method in our armoury is only slightly more difficult. It is called the Sine Rule.

In a triangle *ABC*, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Proof:

Consider a triangle *ABC* in which there is no right angle – meaning that we cannot use our standard trig ratios.



A line drawn from *C*, perpendicular to *AB*, will divide triangle *ABC* into two right-angled triangles, *CDA* and *CDB*. We can use standard ratios in these.

In $\triangle CDA$ $\qquad \sin A = {}^{h}\!/_{b} \quad \Rightarrow \quad h = b \sin A$

In $\triangle CDB$ $\qquad \sin B = {}^{h}\!/_{a} \quad \Rightarrow \quad h = a \sin B$

Therefore $\qquad a \sin B = b \sin A$

i.e. $\qquad \frac{a}{\sin A} = \frac{b}{\sin B}$

We could equally well have divided $\triangle ABC$ into two right-angled triangles by drawing the perpendicular from *A* to *BC* (or from *B* to *AC*). This would have led to a similar result.

i.e. $\qquad \frac{a}{\sin A} = \frac{b}{\sin B}$

Sine Rule

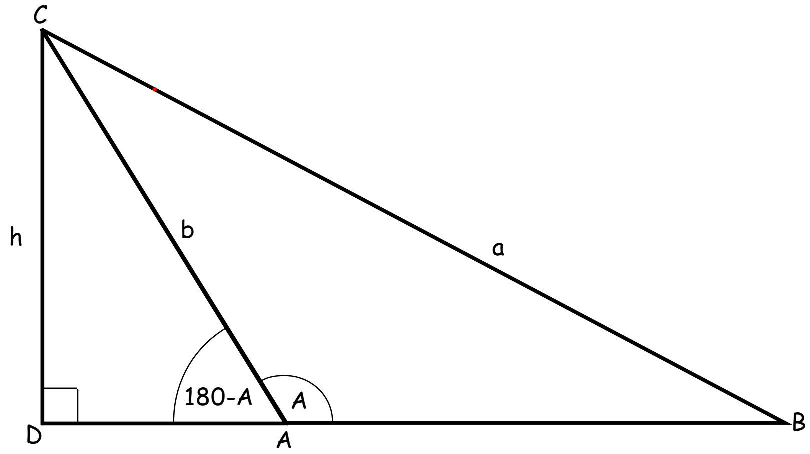
By combining the two results we produce the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This proof is equally valid when $\triangle ABC$ contains an obtuse angle.

Reminder: When working with a triangle ABC the side opposite to $\angle A$ is denoted by a, the side opposite to $\angle B$ by b and so on.

Suppose that $\angle A$ is obtuse.



This time $h = b \sin (180° - A)$ but, as $\sin (180° - A) = \sin A$, we see that once again $h = b \sin A$.

In all other respects the proof given above is unaltered, showing that the sine rule applies to any triangle.

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This rule is made up of three separate fractions, only two of which are used at a time. We select the two which contain three known quantities and only one unknown.

Note that, when the sine rule is being used to find an unknown angle, it is more conveniently written in the form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Student: Date:

Sine Rule

Example: In $\triangle ABC$, BC=5cm, A=43° and B=61° . Find AC.

A, B and a are known and b is required so we use $\frac{a}{\sin A} = \frac{b}{\sin B}$

Substituting $\frac{5}{\sin 43} = \frac{b}{\sin 61}$ so $b = \frac{5\sin 61}{\sin 43} = 6.412$

AC = 6.41 to 3 sig fig.

Now, on a separate sheet of paper, try these questions. Where necessary, give your answers correct to 3 s.f. It will be easier if you do a neat sketch of each triangle. Include this as part of your working.

1) In $\triangle DEF$, DE = 174cm, $\angle D$ = 48° and $\angle F$ = 56°. Find EF.

2) In $\triangle ABC$, AB = 9cm, $\angle A$ = 51° and $\angle C$ = 39°. Find BC.

3) In $\triangle XYZ$, $\angle X$ = 27°, YZ = 6.5cm and $\angle Y$ = 73°. Find ZX.

4) In $\triangle PQR$, $\angle R$ = 52°, $\angle Q$ = 79° and PR = 12.7cm. Find PQ.

5) In $\triangle ABC$, AC = 9.1cm, $\angle A$ = 59° and $\angle B$ = 62°. Find BC.

6) In ABC, AC = 17cm, $\angle A$ = 105° and $\angle B$ = 33°. Find AB.

And these? Take care! Give your answers to the nearest ° or to 3 sig.fig.

7) In $\triangle ABC$, $\angle A$ = 35°, BC = 3cm and AB = 5cm. Find $\angle C$.

8) In $\triangle$XYZ, XZ = 11cm, $\angle$Y = 41° and YZ = 8cm. Find $\angle$X.

9) In $\triangle ABC$, $\angle$B = 40°, BC = 2.9cm and AC = 6.1cm. Find $\angle A$.

10) In $\triangle$XYZ, XY = 5.7cm, $\angle$Y = 20° and XZ = 2.3cm. Find $\angle$Z.

11) In $\triangle ABC$, $\angle A$ = 29.5°, BC = 36cm and AB = 21cm. Find $\angle C$.

12) In $\triangle$XYZ, XZ = 3.8cm, $\angle$Y = 54° and YZ = 2.7cm. Find $\angle$X.

13) In $\triangle ABC$, $\angle C$ = 33°, AC = 7.1cm and AB = 4.6cm. Find $\angle$B.

14) In $\triangle$XYZ, XY = 9cm, $\angle Z$ = 40° and YZ = 7cm. Find $\angle$X.

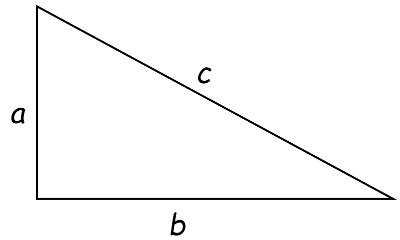
Pythagoras' Theorem

Pythagoras' theorem

Important note: assume throughout this set that all triangles are right angled.

Pythagoras' theorem describes the relationship between the lengths of the sides of a **right-angled** triangle. The longest side of a right-angled triangle is called the **hypotenuse**.

$a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$



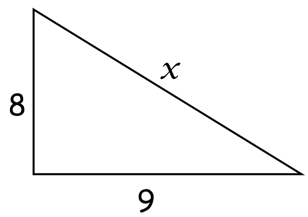
(short side)^2 + (short side)^2 = (hypotenuse)^2

Pythagoras' theorem allows you to find one side of a right-angled triangle if you know the other two sides.

Examples

Find the length x in these triangles. Give your answer to 3 sig.fig.

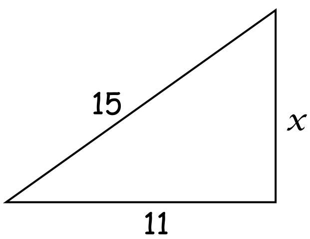
a)



The missing side is the hypotenuse.

$x^2 = 9^2 + 8^2$
$= 81 + 64$
$= 145$
$x = \sqrt{145}$ = 12.0 cm (3 sf)

b)



This time a shorter side is missing.

$15^2 = x^2 + 11^2$
$x^2 = 15^2 - 11^2$
$= 225 - 121$
$= 104$
$x = \sqrt{104}$ = 10.2 cm (3 sf)

Try this question:

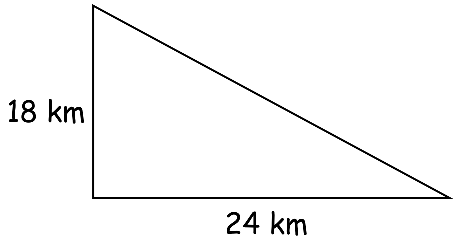
A helicopter flies 24 km west and then 18 km north. How far is the helicopter from its starting position?

Draw a sketch to help your calculation and show all working clearly.

Pythagoras' Theorem

Worked Answer:

Draw a sketch and identify which side is missing.



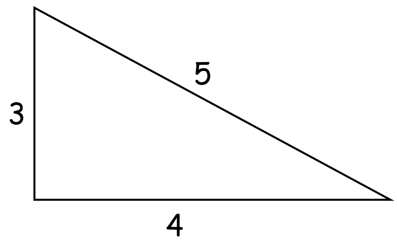
$x^2 = 24^2 + 18^2 = 900$

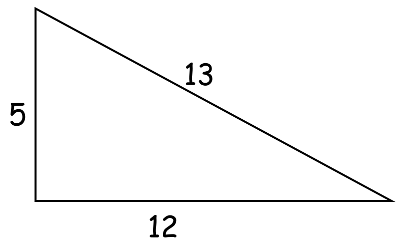
$x = \sqrt{900} = 30$ km

The helicopter is 30km from its starting position.

Standard triangles

Two well-known "standard" right-angled triangles are the 3, 4, 5 triangle and the 5, 12, 13 triangle. Any enlargements of these triangles will also be right-angled, for example 6, 8, 10 or 15, 36, 39 etc.





Try this question:

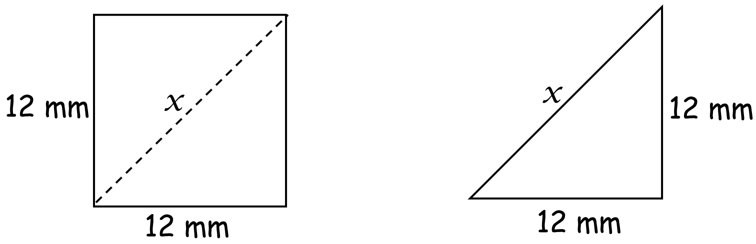
Find the length of the diagonal of a square of side length 12 mm. Give your answer to 3 significant figures.

Draw a sketch to help your calculation and show all working clearly.

Pythagoras' Theorem

Worked Answer:

Draw the square and label the diagonal x.



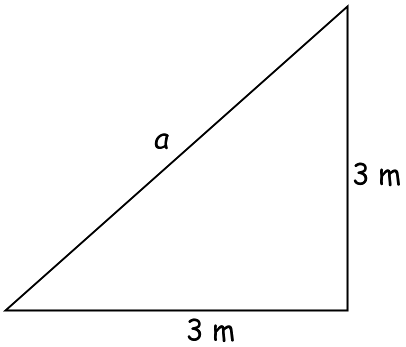
$x^2 = 12^2 + 12^2 = 288$

$x = \sqrt{288} = 17.0$ mm (3 sf)

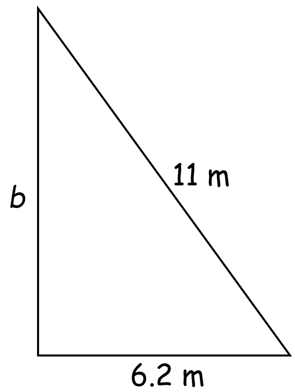
The diagonal of a square of side 12mm is 17.0 mm (to 3 sf)

Find the missing sides in these triangles, giving your answers to 3sf.

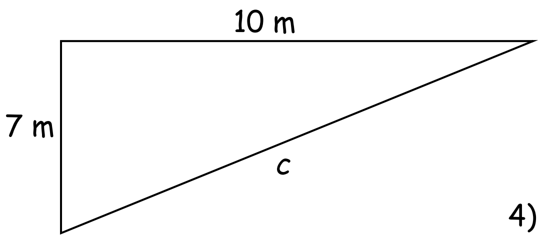
1)



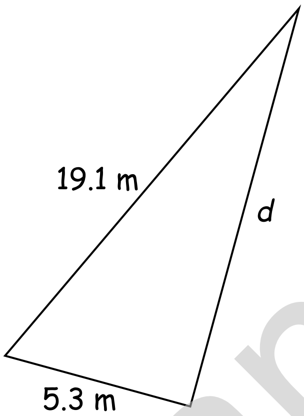
2)



3)



4)



Student: Date:

Pythagoras' Theorem

Try these questions, writing all working out clearly and drawing sketch diagrams when it helps. Use lined paper or your notebook.

5) Janet puts some wire fencing around this triangular garden. She uses 5 m for the shorter side and 12 m for the longest side. What is the total length of wire that she uses?

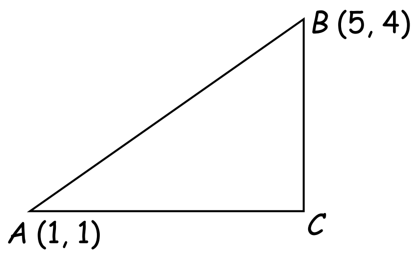
6) Mark walks 400 m east and then 250 m south. How far is Mark from his starting point?

7) *A* and *B* have coordinates (1, 1) and (5, 4) respectively.

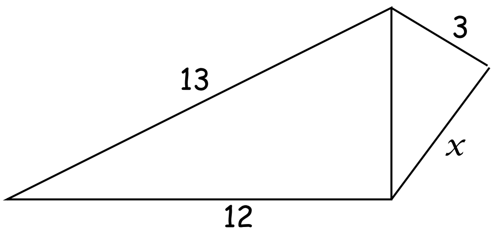
a) What is the length of *AC*?

b) What is the length of *BC*?

c) Calculate the length of *AB*.



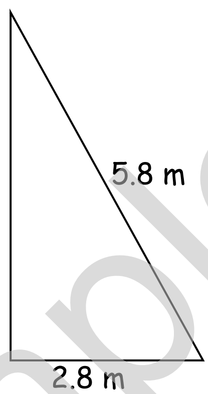
8) Find the length x in this diagram.



9) A sail has the shape shown in the diagram. Reinforcing tape is put around its edges.

a) Assuming the sail is approximately a right-angled triangle, how much tape is needed in total?

b) If the tape costs 87 pence per metre, what will be the cost (answer in £ p).



10) Do you think 7 cm, 19 cm and 15 cm could be the lengths of the sides of a right-angled triangle? Explain your answer carefully.

